BILATERAL WEIGHTED SHIFT OPERATORS
SIMILAR TO NORMAL OPERATORS

György Pál Gehér

Abstract. We prove that an injective, not necessarily bounded weighted bilateral shift operator on $\ell^2(\mathbb{Z})$ is similar to a normal operator if and only if it is similar to a scalar multiple of the simple (i.e. unweighted) bilateral shift operator S.

REFERENCES

[1] N. E. BENAMARA AND N. NIKOLSKI, *Resolvent tests for similarity to a normal operator*, Proc. London Math. Soc. **78**, no. 3 (1999), 585–626.

[2] J. B. CONWAY, *A Course in Functional Analysis*, Springer–Verlag, New York, 1990.

[3] S. R. FOGUEL, *A counterexample to a problem of Sz.-Nagy*, Proc. Amer. Math. Soc. **15**, no. 5 (1964), 788–790.

[4] GY. P. GEHÉR, *Asymptotic limits of operators similar to normal operators*, Proc. Amer. Math. Soc. **143**, 11 (2015), 4823–4834.

[5] P. R. HALMOS, *Ten problems in Hilbert space*, Bull. Amer. Math. Soc. **76**, no. 5 (1970), 887–933.

[6] C. S. KUBRUSLY, *An Introduction to Models and Decompositions in Operator Theory*, Birkhäuser, Boston, 1997.

[7] S. KUPIN, *Linear resolvent growth test for similarity of a weak contraction to a normal operator*, Ark. Mat. **39**, no. 1 (2001), 95–119.

[8] S. KUPIN, *Operators similar to contractions and their similarity to a normal operator*, Indiana Univ. Math. J. **52**, no. 3 (2003), 753–768.

[9] A. LEBOW, *A power-bounded operator that is not polynomially bounded*, Michigan Math. J. **15**, no. 4 (1968), 397–399.

[10] N. K. NIKOLSKI, *Treatise on the Shift Operator*, Springer–Verlag, Berlin, 1986.

[11] G. PISIER, *A polynomially bounded operator on Hilbert space which is not similar to a contraction*, J. Amer. Math. Soc. **10**, no. 2 (1997), 351–369.

[12] S. ÔTA AND K. SCHMÜDGEN, *On some classes of unbounded operators*, Integr. Equat. Oper. Th. **12**, no. 2 (1989), 211–226.

[13] H. N. SALAS, *Hypercyclic weighted shifts*, Trans. Amer. Math. Soc. **347**, no. 3 (1995), 993–1004.

[14] H. N. SALAS, *Supercyclicity and weighted shifts*, Studia Math. **135**, no. 1 (1999), 55–74.

[15] A. L. SHIELDS, *Weighted shift operators and analytic function theory*, Topics in Operator Theory, Math. Surveys 13, Amer. Math. Soc., Providence, R. I. (1974), 49–128.

[16] B. SZ.-NAGY, C. FOIAS, H. BERCOVICI AND L. KÉRCHY, *Harmonic Analysis of Operators on Hilbert Space*, Springer, New York, 2010.

[17] B. SZ.-NAGY, *On uniformly bounded linear transformations in Hilbert space*, Acta Sci. Math. (Szeged) **11**, no. 3–3 (1947), 152–157.

Operators and Matrices
www.ele-math.com
oam@ele-math.com