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FINITE RANK PERTURBATIONS AND SOLUTIONS TO THE OPERATOR RICCATI EQUATION

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Abstract. We consider an off-diagonal self-adjoint finite rank perturbation of a self-adjoint operator in a complex separable Hilbert space $\mathfrak{H}_0 \oplus \mathfrak{H}_1$, where $\mathfrak{H}_1$ is finite dimensional. We describe the singular spectrum of the perturbed operator and establish a connection with solutions to the operator Riccati equation. In particular, we prove existence results for solutions in the case where the whole Hilbert space is finite dimensional.

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