LOG AND HARMONICALLY LOG–CONVEX
FUNCTIONS RELATED TO MATRIX NORMS

Mohammad Sababheh

Abstract. In this article, we introduce the concept of harmonically log-convex functions, which seems to be strongly connected to unitarily invariant norms. Then, we prove Hermite-Hadamard inequalities for these functions. As an application, we present many inequalities for the trace operator and unitarily invariant norms.

REFERENCES

[1] R. BHATIA, *Matrix analysis*, Springer-Verlag, New York, 1997.

[2] J. BOURIN, *Some inequalities for norms on matrices and operators*, Linear Algebra Appl. **292** (1999) 139–154.

[3] E. CARLEN, *Trace Inequalities and Quantum Entropy: An Introductory Course*, Contemp. Math. **529** (2009).

[4] C. CONDE, *Young type inequalities for positive operators*, Ann. Funct. Anal. **4** (2013) no. 2, 144–152.

[5] S. S. DRAGOMIR, B. MOND, *Integral inequalities of Hadamard type for log-convex functions*, Demonstratio Mathematica, **31** (1998) no. 2, 354–364.

[6] Y. FENG, *Refinements of the Heinz inequalities*, Journal of Inequalities and Applications, 2012:18. Doi:10.1186/1029-242X-2012-18.

[7] F. HIAI, H. KOSAKI, *Comparison of various means for operators*, J. Funct. Anal. **163** (1999), 300–323.

[8] F. HIAI, X. ZHAN, *Inequalities involving unitarily invariant norms and operator monotone functions*, Linear Algebra Appl. **341** (2002), 151–169.

[9] I. ISCAN, S. WU, *Hermite-Hadamard type inequalities for harmonically convex functions via fractional integrals*, Applied Mathematics and Computation, **238** (2014), 237–244. doi:10.1016/j.amc.2014.04.020.

[10] R. KAUR, M. MOSLEHIAN, M. SINGH, C. CONDE, *Further refinements of the Heinz inequality*, Linear Algbera Appl. **447** (2013), 26–37. http://dx.doi.org/10.1016/j.laa.2013.01.012.

[11] F. KITTANEH, *Norm inequalities for fractional powers of positive operators*, Lett. Math. Phys. **27** (1993), 279–285.

[12] F. KITTANEH, *On the convexity of the Heinz mean*, Integr. Equ. Oper. Theory, **68** (2010), 519–527.

[13] M. A. NOOR, K. I. NOOR AND M. U. AWAN, *Some characterization of harmonically log-convex functions*, Proc. Jangjeon Math. Soc. **17**, 1 (2014), 51–61.

[14] M. SABABHEH, A. YOUSF, R. KHALIL, *Interpolated Young and Heinz inequalities*, Linear and Multilinear Algebra, **63** (2015) no. 11, 2232–2244. doi:10.1080/03081087.2014.1000816.

[15] M. SABABHEH, *Interpolated inequalities for unitarily invariant norms*, Linear Algebra Appl., **475** (2015), 240–250. doi:10.1016/j.laa.2015.02.026.

[16] M. SABABHEH, *Integral Inequalities of the Heinz Means as Convex Functions*, J. Math. Inequal., **10**, 2 (2016), 313–325.

Operators and Matrices
www.ele-math.com
oam@ele-math.com