VARIATIONAL PRINCIPLES FOR SELF–ADJOINT OPERATOR FUNCTIONS ARISING FROM SECOND–ORDER SYSTEMS

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Abstract. Variational principles are proved for self-adjoint operator functions arising from variational evolution equations of the form

\[ \langle \ddot{z}(t), y \rangle + \partial \dot{z}(t), y \rangle + a_0[z(t), y] = 0. \]

Here \( a_0 \) and \( \partial \) are densely defined, symmetric and positive sesquilinear forms on a Hilbert space \( H \). We associate with the variational evolution equation an equivalent Cauchy problem corresponding to a block operator matrix \( \mathcal{A} \), the forms

\[ t(\lambda)[x,y] := \lambda^2 \langle x,y \rangle + \lambda \partial[x,y] + a_0[x,y], \]

where \( \lambda \in \mathbb{C} \) and \( x,y \) are in the domain of the form \( a_0 \), and a corresponding operator family \( T(\lambda) \). Using form methods we define a generalized Rayleigh functional and characterize the eigenvalues above the essential spectrum of \( \mathcal{A} \) by a min-max and a max-min variational principle. The obtained results are illustrated with a damped beam equation.


Keywords and phrases: Block operator matrices, variational principle, operator function, second-order equations, spectrum, essential spectrum, sectorial form.

REFERENCES