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POLYNOMIAL AS A NEW VARIABLE — A BANACH ALGEBRA WITH A FUNCTIONAL CALCULUS

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Abstract. Given any square matrix or a bounded operator A in a Hilbert space such that $p(A)$ is normal (or similar to normal), we construct a Banach algebra, depending on the polynomial p, for which a simple functional calculus holds. When the polynomial is of degree d, then the algebra deals with continuous $\mathbb{C}^d$-valued functions, defined on the spectrum of $p(A)$. In particular, the calculus provides a natural approach to deal with nontrivial Jordan blocks and one does not need differentiability at such eigenvalues.

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