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ARROWHEAD OPERATORS ON A HILBERT SPACE

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Abstract. The arrowhead matrices define a class of one-term Sylvester matrix (OTSM) operators on a finite-dimensional Hilbert space through an elementary *UDL* factorization. It enables us to consider the infinite invertible arrowhead matrices *UDL* factored properly for introducing, under suitable conditions, the arrowhead operators and their associated class of OTSM operators on an infinite-dimensional Hilbert space. Properties regarding convergence, inertia, inverses, and spectra are also considered.

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