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ON THE SINGULAR VECTORS OF THE GENERALIZED LYAPUNOV OPERATOR

SHENG CHEN AND YUNBO TIAN

Abstract. In this paper, we study the largest and the smallest singular vectors of the generalized Lyapunov operator. For real matrices A,B with order n, we prove that $\max_{\|X\|_F=1}\|AXB^T+BXA^T\|_F$ is achieved by a symmetric matrix for $n\leqslant 3$ and give a counterexample for order $n=4$. We also prove that $\min_{\|X\|_F=1}\|AXB^T+BXA^T\|_F$ is achieved by a symmetric matrix for $n\leqslant 2$ and give a counterexample for order $n=3$. It is shown that the minimizer is symmetric, if the minimum is zero, or if the real parts of the eigenvalues of $A-\lambda B$ are of one sign.

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REFERENCES

[1] B. D. O. ANDERSON AND J. B. MOORE, *Optimal Control-Linear Quadratic Methods*, Prentice-Hall, Englewood Cliffs, NJ, 1990.
[2] R. H. BARTELS AND G. W. STEWART, *Solution of the equation* $AX+XB=C$, Comm. ACM **15** (1972), pp. 820–826.
[3] P. BENNER AND E. S. QUINTANA-ORTŁ, *Solving stable generalized Lyapunov equations with the matrix sign function*, Numerical Algorithms, **20** (1999), pp. 75–100.
[4] R. BHATIA AND D. DRISSI, *Generalized Lyapunov equations and positive definite functions*, SIAM Journal on Matrix Analysis and Applications, **27** (2005), pp. 103–114.
[5] R. BHATIA, *Matrix analysis*, Springer, 1997.
[6] R. BYERS, *A LINPACK-style condition estimator for the equation* $AX-XB^T=C$, IEEE Trans. Automat. Control., **29** (1984), pp. 926–928.
[7] R. BYERS AND S. NASH, *On the singular "vectors" of the Lyapunov operator*, SIAM J. Algebraic Discrete Methods, **8** (1987), pp. 59–66.
[8] R. BYERS AND D. KRESSNER, *Structured condition numbers for invariant subspaces*, SIAM Journal on Matrix Analysis and Applications, **28** (2006), pp. 326–347.
[9] S. CHEN AND Y. TIAN, *Note on "On the singular "vectors" of the Lyapunov operator" by R. Byers and S. Nash*, SIAM Journal on Matrix Analysis and Applications, **3** (2015), pp. 1069–1072.
[10] D. CHENG, *On Lyapunov mapping and its applications*, Communications in Information and Systems, **1** (2001), pp. 255–272.
[11] D. CHENG, Y. ZHU AND H. QI, *A conjecture on the norm of Lyapunov mapping*, Journal of Control Theory and Applications, **7** (2009), pp. 48–50.
[12] K.-W. E. CHU, *The solution of the matrix equation* $AXB-CXD=Y$ *and* $(YA-DZ,YC-BZ)=(E,F)$, Linear Algebra Appl., **93** (1987), pp. 93–105.
[13] T. DAMM, *Direct methods and ADI-preconditioned Krylov subspace methods for generalized Lyapunov equations*, Numerical Linear Algebra with Applications, **15** (2008), pp. 853–871.
[14] J. D. GARDINER, A. J. LAUB, J. J. AMATO AND C. B. MOLER, *Solution of the Sylvester matrix equation* $AXB^T+CXD^T=E$, ACM Trans. Math. Software, **18** (1992), pp. 223–231.
[15] J. FENG, J. LAM, G. YANG AND Z. LI, *On a conjecture about the norm of Lyapunov mappings*, Linear Algebra and its Applications, **465** (2015), pp. 88–103.
[16] S. J. HAMMARLING, *Numerical solution of the stable non-negative definite Lyapunov equation*, IMA J. Numer. Anal., **2** (1982), pp. 303–323.

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[17] U. HELMKE AND J. B. MOORE, *Optimization and Dynamical Systems*, Springer, London, 1994.

[18] R. A. HORN AND C. R. JOHNSON, *Matrix analysis*, Cambridge university press, 2012.

[19] B. KAGSTROM AND L. WESTIN, *Generalized Schur methods with condition estimators for solving the generalized Sylvester equation*, IEEE Trans. Automat. Control, **34** (1989), pp. 745–751.

[20] M. KONSTANTINOV, V. MEHRMANN AND P. PETKOV, *On properties of Sylvester and Lyapunov operators*, Linear Algebra Appl. **312** (2000), pp. 35–71.

[21] P. LANCASTER AND M. TISMENETSKY, *The Theory of Matrices*, second ed., Academic Press, Orlando, FL, 1985.

[22] W. E. LONGSTAFF, *On tridiagonalization of matrices*, Linear Algebra and its Applications, **109** (1988), pp. 153–163.

[23] V. MEHRMANN, *The Autonomous Linear Quadratic Control Problem, Theory and Numerical Solution, Lecture Notes in Control and Information Sciences*, Vol. **163**, Springer, Heidelberg, 1991.

[24] P. H. PETKOV, N. D. CHRISTOV AND M. M. KONSTANTINOV, *Computational Methods for Linear Control Systems*, Prentice-Hall, Hertfordshire, UK, 1991.

[25] T. PENZL, *Numerical solution of generalized Lyapunov equations*, Advances in Computational Mathematics, **8** (1998), pp. 33–48.

[26] T. STYKEL, *Stability and inertia theorems for generalized Lyapunov equations*, Linear Algebra and its Applications, **355** (2002), pp. 297–314.

[27] W. ZHANG, H. ZHANG AND B. S. CHEN, *Generalized Lyapunov equation approach to state-dependent stochastic stabilization/detectability criterion*, Automatic Control, IEEE Transactions on, **53** (2008), pp. 1630–1642.

[28] W. ZHANG AND B. S. CHEN, *Representation and Applications to Generalized Lyapunov Equations and Linear Stochastic Systems*, Automatic Control, IEEE Transactions on, **57** (2012), pp. 3009–3022.

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