

## ON THE STRUCTURE OF SKEW SYMMETRIC OPERATORS

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*Abstract.* An operator  $T$  on a complex Hilbert space  $\mathcal{H}$  is called skew symmetric if  $T$  can be represented as a skew symmetric matrix relative to some orthonormal basis for  $\mathcal{H}$ . We use multiplicity theory to characterize when there is an anti-conjugation commuting with a fixed positive operator, and give a description of such anti-conjugations. Based on these results, we provide a canonical model of skew symmetric operators in terms of multiplication operators on function spaces.

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