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ON THE GRÜSS INEQUALITY FOR UNITAL 2–POSITIVE LINEAR MAPS

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Abstract. In a recent work, Moslehian and Rajić have shown that the Grüss inequality holds for unital n-positive linear maps $\phi : \mathscr{A} \to B(H)$, where $\mathscr{A}$ is a unital C^*-algebra and H is a Hilbert space, if $n \geqslant 3$. They also demonstrate that the inequality fails to hold, in general, if $n = 1$ and question whether the inequality holds if $n = 2$. In this article, we provide an affirmative answer to this question.

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