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INCLUSION SYSTEMS OF HILBERT MODULES OVER THE C^*–ALGEBRA OF COMPACT OPERATORS

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Abstract. In this note we observe inclusion systems of Hilbert modules over the C^*-algebra of all compact operators acting on a Hilbert space. We prove that if each Hilbert C^*-module in the generated product system is strictly complete, then it is possible to construct a bijection between the set of all units of an inclusion system and a quotient (by a suitable equivalence relation) of a certain set of units in the generated product system. Thereby we obtain a generalization of the result that provides the existence of a bijection between the set of all units in an inclusion system of Hilbert spaces and the set of all units in the generated product system (B. V. R. Bhat and M. Mukherjee [*Inclusion systems and amalgamated products of product systems*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **13** (2010), no. 1, 1–26]).

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REFERENCES

[1] D. BAKIĆ AND B. GULJAŠ, *Operators on Hilbert H^* -modules*, J. Operator Theory **46** (2001), 123–137.
[2] D. BAKIĆ AND B. GULJAŠ, *Hilbert C^*-modules over C^*-algebras of compact operators*, Acta. Sci. Math. (Szeged) **68** (2002), 249–269.
[3] D. BAKIĆ AND B. GULJAŠ, *Extensions of Hilbert C^*-modules*, Houston Journal of Mathematics, Vol. **30** (2), (2004), 537–558.
[4] D. BAKIĆ, *A class of strictly complete Hilbert*, C^*-modules, manuscript (2005).
[5] S. D. BARRETO, B. V. R. BHAT, V. LIEBSCHER AND M. SKEIDE, *Type I product systems of Hilbert modules*, J. Funct. Anal. **212** (2004), 121–181.
[6] B. V. R. BHAT AND M. MUKHERJEE, *Inclusion systems and amalgamated products of product system*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. Vol. **13** (1), (2010), 1–26.
[7] B. V. R. BHAT AND M. SKEIDE, *Tensor product systems of Hilbert modules and dilations of completely positive semigroups*, Infin. Dimens. Anal. Quantum Probab. Relat. Top. **3** (2000), 519–575.
[8] M. FRANK AND V. I. PAULSEN, *Injective and projective Hilbert C^*-modules and C^*-algebras of compact operators*, arXiv:math/0611349v2 [math.OA] (2008).
[9] E. C. LANCE, *Hilbert C^*-Modules: A toolkit for operator algebraists*, Cambridge University Press (1995).
[10] B. MAGAJNA, *Hilbert C^*-modules in which all closed submodules are complemented*, Proceedings of the American Mathematical Society Vol. **125** (3), (1997), 849–852.
[11] V. M. MANUILOV AND E. V. TROITSKY, *Hilbert C^*-Modules*, American Mathematical Society (2005).
[12] M. SKEIDE, *Dilation theory and continuous tensor product systems of Hilbert modules*, PQQP: Quantum Probability and White Noise Analysis XV (2003), World Scientific.
[13] M. SKEIDE, *Hilbert modules and application in quantum probability*, Habilitationsschrift, Cottbus (2001).

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