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A CANONICAL FORM FOR H–UNITARY MATRICES

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Abstract. In this paper matrices A are considered that have the property that $A^*HA = H$, where $H = H^*$ is invertible. A canonical form is given for the pair of matrices (A,H) under transformations $(A,H) \to (S^{-1}AS, S^*HS)$, where S is invertible, in which the canonical form for the A-matrix is the usual Jordan canonical form. The real case is studied as well.

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