Operators and Matrices
Volume 10, Number 4 (2016), 785–828
doi:10.7153/oam-10-47

MEROMORPHIC MATRIX TRIVIALIZATIONS OF FACTORS OF AUTOMORPHY OVER A RIEMANN SURFACE

JOSEPH A. BALL, KEVIN F. CLANCEY AND VICTOR VINNIKOV

Abstract. It is a consequence of the Jacobi Inversion Theorem that a line bundle over a Riemann surface M of genus g has a meromorphic section having at most g poles, or equivalently, the divisor class of a divisor over M contains a divisor having at most g poles (counting multiplicities). We explore various analogues of these ideas for vector bundles and associated matrix divisors over M. The most explicit results are for the genus 1 case. We also review and improve earlier results concerning the construction of automorphic or relatively automorphic meromorphic matrix functions having a prescribed null/pole structure.

Mathematics subject classification (2010): 14D20, 30F10, 30F35, 47A48.
Keywords and phrases: holomorphic vector bundle, factor of automorphy, zero/pole interpolation problems, Abel-Jacobi map, theta functions, transfer-function realization.

REFERENCES

[1] DANIEL ALPAY AND VICTOR VINNIKOV, *Finite dimensional de Branges Spaces on Riemann surfaces*, J. Funct. Anal. **189** (2002), 283–324.
[2] MICHAEL ATIYAH, *Vector bundles over an elliptic curve*, Proc. London Math. Soc. (3) **7** (1957), 414–452.
[3] JOSEPH A. BALL, *Operator extremal problems, expectation operators and applications to operators on multiply connected domains*, J. Operator Theory **1** (1979), 153–175.
[4] JOSEPH A. BALL AND KEVIN F. CLANCEY, *Interpolation with meromorphic matrix functions*, Proc. Amer. Math. Soc. **121** (1994) no. 2, 491–496.
[5] JOSEPH A. BALL, KEVIN F. CLANCEY AND VICTOR VINNIKOV, *Concrete interpolation of meromorphic matrix functions on Riemann surfaces*, in: Interpolation Theory, Systems Theory and Related Topics (Tel Aviv/Rehovot, 1999), Oper. Theory Adv. Appl. **134**, pp. 137–156, Birkhäuser, Basel, 2002.
[6] JOSEPH A. BALL, ISRAEL GOHBERG AND LEIBA RODMAN, *Interpolation of Rational Matrix Functions*, Oper. Theory Adv. Appl. **45**, Birkhäuser, Basel, 1990.
[7] JOSEPH A. BALL AND ANDRÉ C. M. RAN, *Local inverse spectral problems for rational matrix functions*, Integral Equations and Operator Theory **10** (1987) no. 3, 349–415.
[8] JOSEPH A. BALL AND ANDRÉ C. M. RAN, *Global inverse spectral problems for rational matrix functions*, Linear Algebra Appl. **86** (1987), 237–282.
[9] JOSEPH A. BALL AND VICTOR VINNIKOV, *Zero-pole interpolation for meromorphic matrix functions on an algebraic curve and transfer functions of 2D systems*, Acta Applicandae Mathematicae **45** (1996), 239–316.
[10] JOSEPH A. BALL AND VICTOR VINNIKOV, *Zero-pole interpolation for matrix meromorphic functions on a compact Riemann surface and a matrix Fay trisecant identity*, Amer. J. Math **121** no. 4 (1999), 841–888.
[11] CLIFFORD J. EARLE AND ALBERT MARDEN, *Poincaré series with applications to H^p-spaces on bordered Riemann surfaces*, Illinois J. Math. **13** (1969), 274–278.
[12] CLIFFORD J. EARLE AND ALBERT MARDEN, *Projections to automorphic functions*, Proc. Amer. Math. Soc. **19** (1968), 274–278.

Paper OaM-10-47

[13] HERSHEL M. FARKAS AND IRWIN KRA, *Riemann Surfaces*, Graduate Texts in Mathematics **71** (Second Edition), Springer-Verlag, New York, 1992.
[14] JOHN D. FAY, *Theta Functions on Riemann Surfaces*, Lecture Notes in Mathematics **352**, Springer, 1973.
[15] FRANK FORELLI, *Bounded holomorphic functions and projections*, Illinois J. Math. **10** (1966), 367–380.
[16] OTTO FORSTER, *Lectures on Riemann Surfaces*, Graduate Texts in Mathematics **81**, Springer-Verlag, Berlin, 1981.
[17] HANS GRAUERT, *Analytische Faserungen über holomorph-vollständigen Räumen*, Math. Ann. **135** (1958), 263–273.
[18] ROBERT C. GUNNING, *Lectures on Vector Bundles over Riemann Surfaces*, University of Tokyo Press, Tokyo, 1967.
[19] ROBERT C. GUNNING, *Riemann Surfaces and Generalized Theta Functions*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 91, Springer-Verlag, Berlin, 1976.
[20] V. E. KATSNELSON, *Right and left joint system representation of a rational matrix function in general position (system representation theory for dummies)*, in: Operator Theory, System Theory and Related Topics (Beer-Sheva/Rehovot, 1997), Oper. Theory Adv. Appl., **123**, pp. 337–400, Birkhäuser, Basel, 2001.
[21] RICK MIRANDA, *Algebraic Curves and Riemann Surfaces*, Graduate Studies in Mathematics **5**, American Mathematical Society, Providence, 1995.
[22] DAVID MUMFORD (with the collaboration of C. MUSILI, M. NORI, E. PREVIATO, M. STILLMAN AND H. UMENMURA), *Tata Lectures on Theta I*, Progress in Mathematics **28**, Birkhäuser-Boston, 1983; third printing 1994.
[23] DAVID MUMFORD (with the collaboration of C. MUSILI, M. NORI, E. PREVIATO, M. STILLMAN AND H. UMENMURA), *Tata Lectures on Theta II*, Progress in Mathematics **43**, Birkhäuser-Boston, 1984; third printing 1993.
[24] SAMUEL SCHECHTER, *On the inversion of certain matrices*, Mathematical Tables and Other Aids to Computation **13** (66) (1959), 73–77.
[25] ELIAS M. STEIN AND RAMI SHAKARCHI, *Complex Analysis*, Princeton Lectures in Analysis II, Princeton University Press, Princeton and Oxford, 2003.
[26] ANDRÉ WEIL, *Généralisation des fonctions abéliennes*, J. Math. Pures Appl. (9) **17** (1938), 47–87.