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DILATIONS AND CONSTRAINED ALGEBRAS

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Abstract. It is well known that contractive representations of the disk algebra are completely contractive. The Neil algebra $\mathscr{A}$ is the subalgebra of the disk algebra consisting of those functions f for which $f'(0) = 0$. There is a complete isometry from the algebra $R(W)$ of rational functions with poles off of the distinguished variety $W = \{(z,w) : z^2 = w^3,\ |z| < 1\}$ to $\mathscr{A}$. We prove that there are contractive representations of $\mathscr{A}$ which are not completely contractive, and furthermore provide a Kaiser and Varopoulos inspired example of a representation π of $R(W)$ whereby $\pi(z)$ and $\pi(w)$ are contractions, yet π is not contractive. We also present a characterization of those contractive representations of $R(W)$ that are completely contractive. Finally, we show that by contrast, for the variety $\mathscr{V} = \{(z,w) : z^2 = w^2,\ |z| < 1\}$, all contractive representations of the algebra $R(\mathscr{V})$ of rational functions with poles off $\mathscr{V}$ are completely contractive, and we as well provide a simplified proof of Agler's analogous result over an annulus.

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