CRAWFORD NUMBERS OF COMPANION MATRICES

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Abstract. The (generalized) Crawford number $C(A)$ of an $n$-by-$n$ complex matrix $A$ is, by definition, the distance from the origin to the boundary of the numerical range $W(A)$ of $A$. If $A$ is a companion matrix
\[
\begin{bmatrix}
0 & 1 & & \\
& 0 & 1 & \\
& & \ddots & \\
& & & 0 & 1 \\
-an & -an-1 & \cdots & -a2 & -a1
\end{bmatrix},
\]
then it is easily seen that $C(A) \geq \cos(\pi/n)$. The main purpose of this paper is to determine when the equality $C(A) = \cos(\pi/n)$ holds. A sufficient condition for this is that the boundary of $W(A)$ contains a point $\lambda$ for which the subspace of $\mathbb{C}^n$ spanned by the vectors $x$ with $\langle Ax, x \rangle = \lambda \|x\|^2$ has dimension 2, while a necessary condition is \[
\sum_{j=0}^{n-2} a_{n-j} e^{i(n-j)\theta} \sin \left( (j+1)\frac{\pi}{n} \right) = \sin(\pi/n)
\]
for some real $\theta$. Examples are given showing that in general these conditions are not simultaneously necessary and sufficient. We then prove that they are if $A$ is (unitarily) reducible. We also establish a lower bound for the numerical radius $w(A)$ of $A$: $w(A) \geq \cos(\pi/(n+1))$, and show that the equality holds if and only if $A$ is equal to the $n$-by-$n$ Jordan block.


Keywords and phrases: Companion matrix, numerical range, Crawford number.

REFERENCES


