SOME GRÜSS' TYPE INEQUALITIES FOR
TRACE OF OPERATORS IN HILBERT SPACES

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Abstract. Some inequalities of Grüss' type for trace of operators in Hilbert spaces, under suitable assumptions for the involved operators, are given.

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