MINKOWSKI PRODUCT OF CONVEX SETS 
AND PRODUCT NUMERICAL RANGE

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Abstract. Let \( K_1, K_2 \) be two compact convex sets in \( \mathbb{C} \). Their Minkowski product is the set \( K_1 K_2 = \{ \lambda a + \mu b : \lambda \in K_1, \mu \in K_2 \} \). We show that the set \( K_1 K_2 \) is star-shaped if \( K_1 \) is a line segment or a circular disk. Examples for \( K_1 \) and \( K_2 \) are given so that \( K_1 K_2 \) are triangles (including interior) and \( K_1 K_2 \) is not star-shaped. This gives a negative answer to a conjecture by Puchala et. al concerning the product numerical range in the study of quantum information science. Additional results and open problems are presented.

Keywords and phrases: Convex sets, Minkowski product, numerical range.

REFERENCES