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THE ELLIS–GOHBERG INVERSE PROBLEM FOR MATRIX–VALUED WIENER FUNCTIONS ON THE LINE

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Abstract. This paper deals with the Ellis-Gohberg inverse problem for matrix-valued Wiener functions on the line, instead of on the circle, as was done in [4] for scalar functions and in [14] for matrix-valued functions. The problem is reduced to a linear finite matrix equation of which the right hand side is described explicitly in terms of one of the given functions. The results obtained parallel and extend those derived in [14] for Wiener functions on the circle. Special attention is paid to the case when the given functions are Fourier transforms of functions of finite support. In the final section the results are specified further for the case when the given functions are rational matrix functions.

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