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INDECOMPOSABLE MATRICES DEFINING PLANE CUBICS

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Abstract. In this article we find all (decomposable and indecomposable) 6×6 linear determinantal representations of Weierstrass cubics. As a corollary we verify the Kippenhahn conjecture for M_6.

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