

GENERALIZED γ -GENERATING MATRICES AND NEHARI-TAKAGI PROBLEM

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Abstract. Let $\Gamma(f)$ be the block Hankel matrix of negative Fourier coefficients of a matrix valued function (mvf) $f \in L_{\infty}^{p \times q}(\mathbb{T})$ defined on the unit circle \mathbb{T} . In the present paper a matrix Nehari-Takagi problem is considered: Given a Hankel matrix Γ and $\kappa \in \mathbb{N} \cup \{0\}$ find a mvf $f \in L_{\infty}^{p \times q}(\mathbb{T})$, such that $\|f\|_{\infty} \leq 1$ and $\text{rank}(\Gamma(f) - \Gamma) \leq \kappa$. Under certain mild assumption, we establish a one-to-one correspondence between solutions of the Nehari-Takagi problem and solutions of some Takagi-Sarason interpolation problem. The resolvent matrix of the Nehari-Takagi problem is shown to belong to the class of so-called generalized γ -generating matrices, which is introduced and studied in the paper.

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