

WEAVING FRAMES

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Abstract. We study an intriguing question in frame theory we call *Weaving Frames* that is partially motivated by preprocessing of Gabor frames. Two frames $\{\varphi_i\}_{i \in I}$ and $\{\psi_i\}_{i \in I}$ for a Hilbert space \mathbb{H} are *woven* if there are constants $0 < A \leq B$ so that for every subset $\sigma \subset I$, the family $\{\varphi_i\}_{i \in \sigma} \cup \{\psi_i\}_{i \in \sigma^c}$ is a frame for \mathbb{H} with frame bounds A, B . Fundamental properties of woven frames are developed and key differences between weaving Riesz bases and weaving frames are considered. In particular, it is shown that a Riesz basis cannot be woven with a redundant frame. We also introduce an apparently weaker form of *weaving* but show that it is equivalent to weaving. Weaving frames has potential applications in wireless sensor networks that require distributed processing under different frames, as well as preprocessing of signals using Gabor frames.

Mathematics subject classification (2010): 42C15.

Keywords and phrases: Frame, Riesz basis, distance between subspaces.

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