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THE JORDAN ALGEBRAIC STRUCTURE OF THE CIRCULAR CONE

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Abstract. In this paper, we study and analyze the algebraic structure of the circular cone. We establish a new efficient spectral decomposition, set up the Jordan algebra associated with the circular cone, and prove that this algebra forms a Euclidean Jordan algebra with a certain inner product. We then show that the cone of squares of this Euclidean Jordan algebra is indeed the circular cone itself. The circular cones form a much more general class than the second-order cones, so we generalize some important algebraic properties in the Euclidean Jordan algebra of the second-order cones to the Euclidean Jordan algebra of the circular cones.

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