

WEAKLY CLOSED LIE MODULES OF NEST ALGEBRAS

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Abstract. Let $\mathcal{T}(\mathcal{N})$ be a nest algebra of operators on Hilbert space and let \mathcal{L} be a weakly closed Lie $\mathcal{T}(\mathcal{N})$ -module. We construct explicitly the largest possible weakly closed $\mathcal{T}(\mathcal{N})$ -bimodule $\mathcal{J}(\mathcal{L})$ and a weakly closed $\mathcal{T}(\mathcal{N})$ -bimodule $\mathcal{K}(\mathcal{L})$ such that

$$\mathcal{J}(\mathcal{L}) \subseteq \mathcal{L} \subseteq \mathcal{K}(\mathcal{L}) + \mathcal{D}_{\mathcal{K}(\mathcal{L})},$$

$[\mathcal{K}(\mathcal{L}), \mathcal{T}(\mathcal{N})] \subseteq \mathcal{L}$ and $\mathcal{D}_{\mathcal{K}(\mathcal{L})}$ is a von Neumann subalgebra of the diagonal $\mathcal{T}(\mathcal{N}) \cap \mathcal{T}(\mathcal{N})^*$.

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