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ANALYTIC FUNCTIONS ON THE BIDISK AT BOUNDARY SINGULARITIES VIA HILBERT SPACE METHODS

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Abstract. We investigate the behavior of a generalized Hilbert space model of a function in the Schur class of the bidisk at singular boundary points that satisfy a growth condition. We examine the relationship between the boundary behavior of Schur functions and the geometry of corresponding generalized Hilbert space models. We describe a geometric condition on an associated operator that classifies the behavior of the directional derivative of the underlying Schur function at a carapoint.

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