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ON THE DJL CONJECTURE FOR ORDER 6

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Abstract. In 1994 Drew, Johnson and Loewy conjectured that for $n \geqslant 4$, the cp-rank of any $n \times n$ completely positive matrices is at most $\lfloor n^2/4 \rfloor$. Recently this conjecture has been proved for $n = 5$ and disproved for $n \geqslant 7$, leaving the case $n = 6$ open. We make a step toward proving the conjecture for $n = 6$. We show that if A is a 6×6 completely positive matrix that is orthogonal to an exceptional extremal copositive matrix, then the cp-rank of A is at most 9.

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