ANALYTIC MODEL OF DOUBLY COMMUTING CONTRACTIONS

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Abstract. An \( n \)-tuple \((T_1, \ldots, T_n)\), of commuting bounded linear operators on a Hilbert space \( \mathcal{H} \) is doubly commuting if \( T_i T_j^* = T_j T_i \) for all \( 1 \leq i < j \leq n \). If in addition, each \( T_i \in C_0 \), then we say that \( T \) is a doubly commuting pure tuple. In this paper we prove that a doubly commuting pure tuple \( T \) can be dilated to a tuple of shift operators on some suitable vector-valued Hardy space \( H^2_{\mathcal{D}^n} \). As a consequence of the dilation theorem, we prove that there exists a closed subspace \( \mathcal{S}_T \) of the form

\[
\mathcal{S}_T := \sum_{i=1}^n \Phi T_i H^2_{\mathcal{D}^n} \),
\]

such that \( \mathcal{H} \cong \mathcal{S}_T^\perp \) and

\[(T_1, \ldots, T_n) \cong P_{\mathcal{S}_T} (M_{T_1}, \ldots, M_{T_n}) |_{\mathcal{S}_T^\perp},\]

where \( \{ \mathcal{E}_T \}_{i=1}^n \) are Hilbert spaces and each \( \Phi T_i \in H^\infty (\mathcal{E}_T, \mathcal{D}) \), \( 1 \leq i \leq n \) is either a one variable either a one variable inner function in \( z_i \), or the zero function.


Keywords and phrases: Hardy space over polydisc, doubly commuting contractions, shift operators, Sz.-Nagy and Foias model, isometric dilation.

REFERENCES