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ANALYTIC MODEL OF DOUBLY COMMUTING CONTRACTIONS

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Abstract. An n-tuple ($n \geqslant 2$), $T = (T_1, \dots, T_n)$, of commuting bounded linear operators on a Hilbert space $\mathscr{H}$ is doubly commuting if $T_i T_j^* = T_j^* T_i$ for all $1 \leqslant i < j \leqslant n$. If in addition, each $T_i \in C_{\cdot 0}$, then we say that T is a doubly commuting pure tuple. In this paper we prove that a doubly commuting pure tuple T can be dilated to a tuple of shift operators on some suitable vector-valued Hardy space $H^2_{\mathscr{D}_{T^*}}(\mathbb{D}^n)$. As a consequence of the dilation theorem, we prove that there exists a closed subspace $\mathscr{S}_T$ of the form

$$\mathscr{S}_T := \sum_{i=1}^{n} \Phi_{T_i} H^2_{\mathscr{E}_{T_i}}(\mathbb{D}^n),$$

such that $\mathscr{H} \cong \mathscr{S}_T^{\perp}$ and

$$(T_1, \dots, T_n) \cong P_{\mathscr{S}_T^{\perp}}(M_{z_1}, \dots, M_{z_n})|_{\mathscr{S}_T^{\perp}}$$

where $\{\mathscr{E}_{T_i}\}_{i=1}^n$ are Hilbert spaces and each $\Phi_{T_i} \in H^\infty_{\mathscr{B}(\mathscr{E}_{T_i}, \mathscr{D}_{T^*})}(\mathbb{D}^n)$, $1 \leqslant i \leqslant n$ is either a one variable either a one variable inner function in z_i, or the zero function.

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Paper OaM-11-07

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