

A BOUND FOR THE HILBERT–SCHMIDT NORM OF GENERALIZED COMMUTATORS OF NONSELF-ADJOINT OPERATORS

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Abstract. Let A, \tilde{A} and B be bounded linear operators in a Hilbert space, and $f(z)$ be a function regular on the convex hull of the union of the spectra of A and \tilde{A} . Let SN_2 be the ideal of Hilbert-Schmidt operators. In the paper, a sharp estimate for the Hilbert-Schmidt norm of the commutator $f(A)B - Bf(\tilde{A})$ is established, provided $AB - B\tilde{A} \in SN_2$, $A - A^* \in SN_2$ and $\tilde{A} - \tilde{A}^* \in SN_2$. Here the star means the adjointness. Our results are new even in the finite dimensional case.

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