

## DIFFUSIVE SYSTEMS AND WEIGHTED HANKEL OPERATORS

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*Abstract.* We consider diffusive systems, regarded as input/output systems with a kernel given as the Fourier–Borel transform of a measure in the left half-plane. Associated with these are a family of weighted Hankel integral operators, and we provide conditions for them to be bounded, Hilbert–Schmidt or nuclear, thereby generalizing results of Widom, Howland and others.

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