

MAPS PRESERVING THE LOCAL SPECTRUM OF THE SKEW JORDAN PRODUCT OF OPERATORS

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Abstract. Let \mathcal{H} and \mathcal{K} be two infinite-dimensional complex Hilbert spaces, and fix two nonzero vectors $h_0 \in \mathcal{H}$ and $k_0 \in \mathcal{K}$. Let $\mathcal{L}(\mathcal{H})$ (resp. $\mathcal{L}(\mathcal{K})$) denote the algebra of all bounded linear operators on \mathcal{H} (resp. on \mathcal{K}), and let $\mathcal{F}_2(\mathcal{K})$ be the set of all operators in $\mathcal{L}(\mathcal{K})$ of rank at most two. We show that a map φ from $\mathcal{L}(\mathcal{H})$ into $\mathcal{L}(\mathcal{K})$ such that its range contains $\mathcal{F}_2(\mathcal{K})$ satisfies

$$\sigma_{\varphi(T)\varphi(S)^* + \varphi(S)^*\varphi(T)}(k_0) = \sigma_{TS^* + S^*T}(h_0), \quad (T, S \in \mathcal{L}(\mathcal{H})),$$

if and only if there exist a unitary operator U from \mathcal{H} into \mathcal{K} and a scalar $\alpha \in \mathbb{C}$ such that $Uh_0 = \alpha k_0$ and $\varphi(T) = \lambda UTU^*$ for all $T \in \mathcal{L}(\mathcal{H})$, where λ is a scalar of modulus 1.

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