MAPS PRESERVING THE LOCAL SPECTRUM OF THE SKEW JORDAN PRODUCT OF OPERATORS

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Abstract. Let $\mathcal{H}$ and $\mathcal{K}$ be two infinite-dimensional complex Hilbert spaces, and fix two nonzero vectors $h_0 \in \mathcal{H}$ and $k_0 \in \mathcal{K}$. Let $\mathcal{L}(\mathcal{H})$ (resp. $\mathcal{L}(\mathcal{K})$) denote the algebra of all bounded linear operators on $\mathcal{H}$ (resp. on $\mathcal{K}$), and let $\mathcal{F}_2(\mathcal{K})$ be the set of all operators in $\mathcal{L}(\mathcal{K})$ of rank at most two. We show that a map $\varphi$ from $\mathcal{L}(\mathcal{H})$ into $\mathcal{L}(\mathcal{K})$ such that its range contains $\mathcal{F}_2(\mathcal{K})$ satisfies

$$\sigma_{\varphi(T)\varphi(S)^*+\varphi(S)^*\varphi(T)}(k_0) = \sigma_{TS^*+S^*T}(h_0), \quad (T, S \in \mathcal{L}(\mathcal{H})), $$

if and only if there exist a unitary operator $U$ from $\mathcal{H}$ into $\mathcal{K}$ and a scalar $\alpha \in \mathbb{C}$ such that $Uh_0 = \alpha k_0$ and $\varphi(T) = \lambda U T U^*$ for all $T \in \mathcal{L}(\mathcal{H})$, where $\lambda$ is a scalar of modulus 1.


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REFERENCES