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NUMERICAL RANGES OF THE PRODUCT OF OPERATORS

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Abstract. We study containment regions of the numerical range of the product of operators A and B such that $W(A)$ and $W(B)$ are line segments. It is shown that the containment region is equal to the convex hull of elliptical disks determined by the spectrum of AB, and conditions on A and B for the set equality holding are obtained. The results cover the case when A and B are self-adjoint operators extending the previous results on the numerical range of the product of two orthogonal projections.

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