

## BERNSTEIN FUNCTIONS OF SEVERAL SEMIGROUP GENERATORS ON BANACH SPACES UNDER BOUNDED PERTURBATIONS

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*Abstract.* The paper deals with (multidimensional and one-dimensional) Bochner-Phillips functional calculus. Bounded perturbations of Bernstein functions of (one or several commuting) semigroup generators on Banach spaces are considered, conditions for Lipschitzness and Frechet-differentiability of such functions are obtained, estimates for the norm of commutators are proved, and a generalization of Livschits-Kreĭn trace formula derived.

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