

MOORE–PENROSE INVERSE OF CONDITIONAL TYPE OPERATORS

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Abstract. We prove some basic results on some Moore-Penrose inverse of conditional type operators on $L^2(\Sigma)$. For instance, we show, among other results, that a weighted conditional operator $T = M_w E M_u$ is centered if and only if T^\dagger , the Moore-Penrose inverse of T , is centered. In addition, we establish lower and upper bounds for the numerical range of T and T^\dagger .

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