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ON GENERALIZED DERIVATION IN BANACH SPACES

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Abstract. In this paper we generalized two important results of B. P. Duggal [4, Theorem 2.1 and 2.6], and other results are also given. If $B(\mathscr{X})$ is the algebra of all bounded linear operators on a complex Banach space $\mathscr{X}$ and $J(\mathscr{X})=\{x\in B(\mathscr{X}): x=x_1+ix_2$, where x_1 and x_2 are hermitian$\}$, two results of orthogonality in the sense of Birkhoff are shown $\|a+b\|\leqslant\|a+b-[x^*,x]\|$ and $\|ab\|\leqslant\|ab-[xx^*,x^*x]\|$ for all $x\in J(\mathscr{X})\cap\delta_{a,b}^{-1}(0)$. As application of our first result the William's theorem "Any hermitian element is finite element" is also established with a shorter and simpler proof.

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