

## PERTURBATION OF $(m, p)$ -ISOMETRIES BY NILPOTENT OPERATORS AND THEIR SUPERCYCLICITY

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*Abstract.* A bounded linear operator  $T$  on a Hilbert space  $H$  is an  $(m, p)$ -isometry if

$$\sum_{k=0}^m (-1)^k \binom{m}{k} \|T^k x\|^p = 0$$

for all  $x \in H$ , in which  $p \in [1, \infty)$  and  $m \geq 1$ . In this paper, two significant results will be proved. First, we introduce some perturbations of  $(m, p)$ -isometries which are  $(n, p)$ -isometries for some suitable  $n$ . Indeed, we show that the sum of an  $(m, p)$ -isometry and a commuting nilpotent operator of degree  $r$  is a  $(pr - p + m, p)$ -isometry for every even number  $p$ . As an application, the second result is to prove that such operators are not  $N$ -supercyclic for any positive integer  $N$ , even if  $p$  is a rational number. These results generalize the previous works on  $m$ -isometries.

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### REFERENCES

- [1] J. ALGER, M. STANKUS, *m-isometric transformations of Hilbert space I*, Integr. Equ. Oper. Theory **21** (1995), 383–429.
- [2] J. AGLER, M. STANKUS, *m-isometric transformations of Hilbert space II*, Integr. Equ. Oper. Theory **23** (1995), 1–48.
- [3] J. AGLER, M. STANKUS, *m-isometric transformations of Hilbert space III*, Integr. Equ. Oper. Theory **24** (1996), 379–421.
- [4] S. I. ANSARI, P. S. BOURDON, *Some properties of cyclic operators*, Acta Sci. Math. (Szeged), **63** (1997), 195–207.
- [5] A. ATHAVALE, *Some operator theoretic calculus for positive definite kernels*, Proc. Amer. Math. Soc. **112** (3) (1991), 701–708.
- [6] F. BAYART, *m-isometries on Banach spaces*, Math. Nachr. **284** (2011), 2141–2147.
- [7] F. BAYART, E. MATHERON, *Dynamics of Linear Operators*, vol. 179, Cambridge University Press, 2009.
- [8] T. BERMÚDEZ, I. MARRERO, A. MARINÓN, *On the orbit of an m-isometry*, Integr. Equ. Oper. Theory **64** (4) (2009), 487–494.
- [9] T. BERMÚDEZ, A. MARTINÓN, E. NEGRIN, *Weighted shift operators which are m-isometries*, Integr. Equ. Oper. Theory **68** (3) (2010), 301–312.
- [10] T. BERMÚDEZ, A. MARTINÓN, V. MÜLLER AND J. A. NODA, *Perturbation of m-isometries by nilpotent operators*, Abstr. Appl. Anal. **2014** (2014), Article ID 745479, 6 pages.
- [11] T. BERMÚDEZ, A. MARTINÓN AND J. A. NODA, *An isometry plus a nilpotent operator is an m-isometry*, Applications, J. Math. Anal. Appl. **407** (2) (2013), 505–512.
- [12] P. S. BOURDON, N. FELDMAN, J. SHAPIRO, *Some properties of N-supercyclic operators*, Studia Math. **165** (2) (2004), 135–157.

- [13] M. FAGHIH-AHMADI, *Powers of  $A - m$ -isometric operators and their supercyclicity*, Bull. Malays. Math. Sci. Soc. **39** (3) (2016), 901–911.
- [14] M. FAGHIH-AHMADI, K. HEDAYATIAN, *Hypercyclicity and supercyclicity of  $m$ -isometric operators*, Rocky Mountain J. Math., **42** (1) (2012), 15–23.
- [15] N. FELDMAN,  *$n$ -supercyclic operators*, Studia Math. **151** (2) (2002), 141–159.
- [16] K. G. GROSSE-ERDMANN, A. PERIS MANGUILLOT, *Linear Chaos*, Springer-Verlag London Limited, 2011.
- [17] G. GU AND M. STANKUS, *Some results on higher order isometries and symmetries: products and sums with a nilpotent operator*, Linear Algebra Appl. **469** (2015), 500–509.
- [18] H. M. HILDEN, L. J. WALLEN, *Some cyclic and non-cyclic vectors of certain operators*, Indiana Univ. Math. J. **23** (1974), 557–565.
- [19] P. HOFFMANN, M. MACKEY AND M. SEARCOÍD, *On the second parameter of an  $(m, p)$ -isometry*, Integr. Equ. Oper. Theory **71** (2011), 389–405.
- [20] T. LE, *Algebraic properties of operator roots of polynomials*, J. Math. Anal. Appl. **421** (2) (2015), 1238–1246.
- [21] V. G. MILLER, *Remarks on finitely hypercyclic and finitely supercyclic operators*, Integr. Equ. Oper. Theory **29** (1997), 110–115.
- [22] O. A. M. SID AHMED,  *$m$ -isometric operators on Banach spaces*, Asian-European J. Math. **3** (1) (2010), 1–19.
- [23] S. YARMAHMOODI, K. HEDAYATIAN, B. YOUSEFI, *Supercyclicity and hypercyclicity of an isometry plus a nilpotent*, Abstr. Appl. Anal. **2011** (2011), Article ID 686832, 11 pages.