

LIFTING COMMUTING 3-ISOMETRIC TUPLES

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Abstract. An operator T is called a 3-isometry if there exists operators $B_1(T^*, T)$ and $B_2(T^*, T)$ such that

$$Q(n) = T^{*n}T^n = 1 + nB_1(T^*, T) + n^2B_2(T^*, T)$$

for all natural numbers n . An operator J is a Jordan operator of order 2 if $J = U + N$ where U is unitary, N is nilpotent order 2, and U and N commute. An easy computation shows that J is a 3-isometry and that the restriction of J to an invariant subspace is also a 3-isometry. Those 3-isometries which are the restriction of a Jordan operator to an invariant subspace can be identified, using the theory of completely positive maps, in terms of a positivity condition on the operator pencil $Q(s)$. In this article, we establish the analogous result in the multi-variable setting and show, by modifying an example of Choi, that an additional hypothesis is necessary. Lastly we discuss the joint spectrum of sub-Jordan tuples and derive results for 3-symmetric operators as a corollary.

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