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MULTIPLIERS OF HILBERT SPACES OF ANALYTIC FUNCTIONS ON THE COMPLEX HALF–PLANE

ANDRZEJ STANISŁAW KUCIK

Abstract. It follows, from a generalised version of Paley-Wiener theorem, that the Laplace transform is an isometry between certain spaces of weighted L^2 functions defined on $(0,\infty)$ and (Hilbert) spaces of analytic functions on the right complex half-plane (for example Hardy, Bergman or Dirichlet spaces). We can use this fact to investigate properties of *multipliers* and *multiplication operators* on the latter type of spaces. In this paper we present a full characterisation of multipliers in terms of a generalised concept of a *Carleson measure*. Under certain conditions, these spaces of analytic functions are not only Hilbert spaces but also Banach algebras, and are therefore contained within their spaces of multipliers. We provide some necessary as well as sufficient conditions for this to happen and look at its consequences.

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