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CONVEX–CYCLIC MATRICES, CONVEX–POLYNOMIAL INTERPOLATION AND INVARIANT CONVEX SETS

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Abstract. We define a convex-polynomial to be one that is a convex combination of the monomials $\{1, z, z^2, \ldots\}$. This paper explores the intimate connection between peaking convex-polynomials, interpolating convex-polynomials, invariant convex sets, and the dynamics of matrices. In particular, we use these intertwined relations to both prove which matrices are convex-cyclic while at the same time proving that we can prescribe the values and a finite number of the derivatives of a convex-polynomial subject to certain natural constraints. These properties are also equivalent to determining those matrices whose nonempty invariant closed convex sets are all invariant subspaces.

Our characterization of the convex-cyclic matrices gives a new and correct proof of a similar result by Rezaei that was stated and proven incorrectly.

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