INEQUALITIES FOR EIGENVALUES OF
COMPACTLY PERTURBED UNITARY OPERATORS

Michael Gil'

Abstract. We consider the operator $A = U + K$, where U is a unitary operator and K is a compact one. An eigenvalue λ of A is said to be a non-unitary one, if $|\lambda| \neq 1$. We derive inequalities for sums of absolute values of the non-unitary eigenvalues. Applications of these inequalities to operator functions, spectrum perturbations and operator equations are also discussed.

REFERENCES

[1] A. A. BEN, *Fixed points, eigenvalues and surjectivity for* (ws)*-compact operators on unbounded convex sets*, Cent. Eur. J. Math. **11**, (2013), no. 1, 85–93.

[2] A. A. BONDAR, AND YA. M. DYMARSKII, *Submanifolds of compact operators with fixed multiplicities of eigenvalues*, Ukrainian Math. J. **63**, (2012), no. 9, 1349–1360.

[3] YU. L. DALECKII AND M. G. KREIN, *Stability of Solutions of Differential Equations in Banach Space*, Amer. Math. Soc., Providence, R. I, 1971.

[4] M. DEMUTH, M. HANSMANN, AND G. KATRIEL, *On the discrete spectrum of non-selfadjoint operators*, J. Funct. Anal. **257** (9), (2009) 2742–2759.

[5] M. DEMUTH, M. HANSMANN, AND G. KATRIEL, *Eigenvalues of non-selfadjoint operators: a comparison of two approaches*, in Mathematical physics, spectral theory and stochastic analysis, vol. 232 of Oper. Theory Adv. Appl., pages 107–163, Birkhäuser/Springer, Basel, 2013.

[6] M. DEMUTH, F. HANAUSKA, M. HANSMANN, AND G. KATRIEL, *Estimating the number of eigenvalues of linear operators on Banach spaces*, J. Funct. Anal. **268** (4), (2015) 1032–1052.

[7] M. I. GIL', *Operator Functions and Localization of Spectra*, Lecture Notes in Mathematics, vol. 1830, Springer-Verlag, Berlin, 2003.

[8] M. I. GIL', *Lower bounds for eigenvalues of Schatten-von Neumann operators*, J. of Inequalities in Pure and Appl. Mathem **8**, no. 3, Article 66 (2007), 7 pp.

[9] M. I. GIL', *Bounds for eigenvalues of Schatten-von Neumann operators via self-commutators*, Journal of Functional Analysis **267**, (2014) no. 9, 3500–3506.

[10] M. I. GIL', *Inequalities for Eigenvalues of Compact Operators in a Hilbert Space*, Communications in Contemporary Mathematics, online from 13 March, 2015.

[11] I. C. GOHBERG AND M. G. KREIN, *Introduction to the Theory of Linear Nonselfadjoint Operators*, Trans. Mathem. Monographs, vol. 18, Amer. Math. Soc., Providence, R. I., 1969.

[12] M. HANSMANN, *Variation of discrete spectra for non-selfadjoint perturbations of selfadjoint operators*, Integral Equations and Operator Theory **76** (2), (2013) 163–178.

[13] M. HANSMANN, *Perturbation determinants in Banach spaces with an application to eigenvalue estimates for perturbed operators*, Mathematische Nachrichten, online from July, 29, 2015.

[14] T. KATO, *Perturbation Theory for Linear Operators*, Springer-Verlag, Berlin, 1980.

[15] H. KÖNIG, *Eigenvalue distribution of compact operators*, vol. 16 of Operator Theory: Advances and Applications, Birkhäuser Verlag, Basel, 1986.

[16] H. MAJIDIAN E. BABOLIAN, *An interpolation degenerate kernel method for eigenvalue problems of a class of non-compact operators*, Appl. Math. Lett. **23** (2010), no. 8, 825–830.

[17] A. G. MAZKO, *Matrix Equations, Spectral Problems and Stability of Dynamic Systems*, Stability, Oscillations and Optimization of Systems, Scientific Publishers, Cambridge, 2008.

[18] A. PIETSCH, *Eigenvalues and s-Numbers*, Cambridge Univesity Press, Cambridge, 1987.

[19] S. SHKARIN, *Compact operators without extended eigenvalues*, J. Math. Anal. Appl. **332** (2007), no. 1, 455–462.

Operators and Matrices
www.ele-math.com
oam@ele-math.com