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THE SPECTRAL EQUALITY FOR UPPER TRIANGULAR OPERATOR MATRICES WITH UNBOUNDED ENTRIES

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Abstract. Let

$$M_C = \begin{bmatrix} A & C \\ 0 & B \end{bmatrix} : D(M_C) \subset X \times X \to X \times X$$

be a 2×2 unbounded upper triangular operator matrix on the complex Hilbert space $X \times X$. We investigate the conditions under which $\sigma(M_C) = \sigma(A) \cup \sigma(B)$ holds in the diagonally dominant ($D(M_C) = D(A) \times D(B)$) and upper dominant case ($D(M_C) = D(A) \times D(C)$). Some necessary and sufficient conditions are obtained. The results generalize some results of Han, Du, and Barraa in the bounded case.

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