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DENSITY OF A NORMAL SUBGROUP OF THE INVERTIBLES IN CERTAIN MULTIPLIER ALGEBRAS

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Abstract. Let $\mathscr{A}$ be a unital separable simple C*-algebra. Let $GL(\mathscr{M}(\mathscr{A}\otimes\mathscr{K}))$ be the group of invertible elements of the multiplier algebra of the stabilization of $\mathscr{A}$, and let $N \subseteq GL(\mathscr{M}(\mathscr{A}\otimes\mathscr{K}))$ be any (algebraic) normal subgroup that properly contains the scalar invertibles.

Then

$$\overline{N}^{\text{strict}} = \mathscr{M}(\mathscr{A}\otimes\mathscr{K}),$$

where $\overline{N}^{\text{strict}}$ is the closure of N in the strict topology.

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Paper OaM-11-40

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