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OPERATOR MODELS FOR HILBERT LOCALLY C^*–MODULES

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Abstract. We single out the concept of concrete Hilbert module over a locally C^*-algebra by means of locally bounded operators on certain strictly inductive limits of Hilbert spaces. Using this concept, we construct an operator model for all Hilbert locally C^*-modules and, as an application, we obtain a direct construction of the exterior tensor product of Hilbert locally C^*-modules. These are obtained as consequences of a general dilation theorem for positive semidefinite kernels invariant under an action of a $*$-semigroup with values locally bounded operators. As a by-product, we obtain two Stinespring type theorems for completely positive maps on locally C^*-algebras and with values locally bounded operators.

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REFERENCES

[1] G. ALLAN, *On a class of locally convex algebras*, Proc. London Math. Soc. **15** (1965), 399–421.
[2] C. APOSTOL, *b^*-Algebras and their representations*, J. London Math. Soc. **33** (1971), 30–38.
[3] W. B. ARVESON, *The harmonic analysis of automorphism groups*, in Operator Algebras and Automorphisms, Proc. Symp. Pure Math., vol. **38:1**, pp. 199–269, Amer. Math. Soc. 1982.
[4] W. B. ARVESON, *Dilation theory yesterday and today*, in A Glimpse at Hilbert Space Operators, pp. 99–123, Oper. Theory Adv. Appl., vol. **207**, Birkhäuser Verlag, Basel 2010.
[5] S. AY, A. GHEONDEA, *Representations of $*$-semigroups associated to invariant kernels with values continuously adjointable operators*, Integr. Equ. Oper. Theory **87** (2017), 263–307.
[6] D. E. EVANS, J. T. LEWIS, *Dilations of Irreducible Evolutions in Algebraic Quantum Theory*, Dublin Institute for Advanced Studies, Dublin, 1977.
[7] D. GAŞPAR, P. GAŞPAR, N. LUPA, *Dilations on locally Hilbert spaces*, in Topics in Mathematics, Computer Science and Philosophy, pp. 107–122, Presa Universitară Clujeană, Cluj-Napoca 2008.
[8] A. GHEONDEA, *Dilations of some VH-spaces operator valued kernels*, Integr. Equ. Oper. Theory, **74** (2012), 451–479.
[9] A. GROTHENDIECK, *Topological Vector Spaces*, Gordon and Breach, Montreux 1973.
[10] A. YA. HELEMSKII, *Banach and Locally Convex Algebras*, Oxford University Press, New York, 1993.
[11] A. INOUE, *Locally C^*-algebras*, Mem. Fac. Sci. Kyushu Univ. Ser. A, **25** (1971), 197–235.
[12] M. JOIŢA, *Locally von Neumann algebras*, Bull. Math. Soc. Sci. Math. Roumanie (N.S.) **42** (90) (1999), 51–64.
[13] M. JOIŢA, *Strict completely positive maps between locally C^*-algebras and representations on Hilbert modules*, J. London Mat. Soc., **2** (66) (2002), 421–432.
[14] M. JOIŢA, *Tensor products of Hilbert modules over locally C^*-algebras*, Czech. Math. J. **54** (2004), 727–737.
[15] M. JOIŢA, *Hilbert Modules over Locally C^*-Algebras*, Editura Universităţii Bucureşti, Bucureşti 2006.
[16] D. J. KARIA, Y. M. PARMAR, *Operators on locally Hilbert space*, J. Analysis, **23** (2015), 59–73.
[17] G. G. KASPAROV, *Hilbert C^*-modules: theorems of Stinespring and Voiculescu*, J. Operator Theory, **4** (1980), 133–150.
[18] Y. KOMURA, *Some examples on locally convex spaces*, Math. Annalen, **153** (1964), 150–162.

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[19] G. KÖTHE, *Topological Vector Spaces, I*, Second edition, Springer-Verlag, Berlin 1983.
[20] E. C. LANCE, *Hilbert C^*-Modules. A toolkit for operator algebraists*, London Mathematical Society Lecture Note Series, vol. **210**, Cambridge University Press, Cambridge 1995.
[21] A. MALLIOS, *Hermitian K-theory over topological $*$-algebras*, J. Math. Anal. Appl. **106** (1985), 454–539.
[22] G. J. MURPHY, *C^*-Algebras and Operator Theory*, Academic Press, Boston 1990.
[23] G. J. MURPHY, *Positive definite kernels and Hilbert C^*-modules*, Proc. Edinburgh Math. Soc. **40** (1997), 367–374.
[24] K. R. PARTHASARATY, K. SCHMIDT, *Positive-Definite Kernels, Continous Tensor Products and Central Limit Theorems of Probability Theory*, Lecture Notes in Mathematics, vol. **272**, Berlin: Springer-Verlag, 1972.
[25] N. C. PHILLIPS, *Inverse limits of C^*-algebras*, J. Operator Theory, **19** (1988), 159–195.
[26] K. SCHMÜDGEN, *Über LMC*-Algebren*, Math. Nachr. **68** (1975), 167–182.
[27] Z. SEBESTYEN, *Every C^*-seminorm is automatically submultiplicative*, Period. Math. Hun., **10** (1979), 1–8.
[28] W. F. STINESPRING, *Positive functions on C^*-algebras*, Proc. Amer. Math. Soc. **6** (1955), 211–216.
[29] F. H. SZAFRANIEC, *Murphy's positive definite kernels and Hilbert C^*-modules reorganized*, in Banach Center Publications, vol. **89**, pp. 275–295, Warszawa 2010.
[30] D. VOICULESCU, *Dual algebraic structures on operator algebras related to free products*, J. Operator Theory, **17** (1987), 85–98.
[31] YU. I. ZHURAEV, F. SHARIPOV, *Hilbert modules over locally C^*-algebras*, arXiv: 0011053v3 [math.OA].
[32] B. SZ.-NAGY, *Prolongement des transformations de l'espace de Hilbert qui sortent de cet espace*, in Appendice au livre "Leçons d'analyse fonctionnelle" par F. Riesz et B. Sz.-Nagy, pp. 439–573, Akademiai Kiado, Budapest, 1955.