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KSGNS CONSTRUCTION FOR τ–MAPS ON S–MODULES AND $\mathfrak{K}$–FAMILIES

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Abstract. We introduce S-modules, which generalizes the notion of Krein C^*-modules and where a fixed unitary replaces the symmetry of Krein C^*-modules. The representation theory on S-modules is explored and for a given $*$-automorphism α on a C^*-algebra the KSGNS construction for α-completely positive maps is illustrated. An extention of this construction for τ-maps is also achieved, when τ is an α-completely positive map. We prove decomposition theorems for α-CPD-kernels and $\mathfrak{K}$-families.

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