

ASYMPTOTICS OF GENERALIZED VALUE DISTRIBUTION FOR HERGLOTZ FUNCTIONS

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Abstract. Estimates of limiting value distributions for boundary values of Herglotz functions are extended to allow the possibility of value distributions with respect to measures other than Lebesgue measure. We establish a relation between the generalized theory of value distribution and the angle subtended at a point in the upper half-plane, and we carry out an analysis of the corresponding composed Herglotz functions and their measures. The results are applicable to a description of boundary behaviour for the Weyl m -function in Sturm-Liouville theory.

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