THE INVERTIBILITY FOR LINEAR COMBINATIONS OF BOUNDED LINEAR OPERATORS WITH CLOSED RANGE

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Abstract. In this paper, it is given that the sufficient and necessary conditions for the invertibility of linear combinations of bounded linear operators with closed range. Furthermore, some related results are obtained.

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