

## ESTIMATES FOR THE CORONA THEOREM ON $H_{\mathbb{I}}^{\infty}(\mathbb{D})$

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*Abstract.* Let  $\mathbb{I}$  be a proper ideal of  $H^{\infty}(\mathbb{D})$ . We prove the corona theorem for infinitely many generators in the algebra  $H_{\mathbb{I}}^{\infty}(\mathbb{D})$ . This extends the finite corona results of Mortini, Sasane, and Wick [8]. We also provide the estimates for corona solutions. Moreover, we prove a generalized Wolff's Ideal Theorem for this sub-algebra.

*Mathematics subject classification (2010):* Primary 30H50, Secondary, 30H80, 46J20.

*Keywords and phrases:* Corona theorem, Wolff's theorem,  $H^{\infty}(\mathbb{D})$ , ideals.

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