

SPECTRAL SETS FOR NUMERICAL RANGE

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Abstract. We define and study a numerical-range analogue of the notion of spectral set. Among the results obtained are a positivity criterion and a dilation theorem, analogous to those already known for spectral sets. An important difference from the classical definition is the role played in the new definition by the base point. We present some examples to illustrate this aspect.

Mathematics subject classification (2010): 47A12, 47A25.

Keywords and phrases: Numerical range, spectral set, dilation.

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