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CHARACTERIZATION OF TRUNCATED TOEPLITZ OPERATORS BY CONJUGATIONS

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Abstract. Truncated Toeplitz operators are C-symmetric with respect to the canonical conjugation given on an appropriate model space. However, by considering only one conjugation one cannot characterize truncated Toeplitz operators. It will be proved, for some classes of inner functions and the model spaces connected with them, that if an operator on a model space is C–symmetric for a certain family of conjugations in the model space, then is has to be truncated Toeplitz. A characterization of classical Toeplitz operators is also presented in terms of conjugations.

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