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SPECTRUM OF (n,k)–QUASIPARANORMAL OPERATORS

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Abstract. In this work, some spectral properties of (n,k)-quasiparanormal operators are considered. Let T be a (n,k)-quasiparanormal operator, $\mathscr{M}$ a nontrivial closed invariant subspace of T and $T = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix}$ on $\mathscr{M} \oplus \mathscr{M}^{\perp}$. (i) Isolated spectral points and poles. Every nonzero isolated spectral point of T is a pole of order one. (ii) Point spectrum and finite ascent. If $\lambda \neq 0$ and $\mathscr{M} = \ker(T-\lambda) \neq \{0\}$, then $\ker(T_{22}-\lambda) = \{0\}$. Thus $\ker(T-\lambda) = \ker(T-\lambda)^2$. In particular, if λ is nonzero isolated spectral point, then $T_{22}-\lambda$ is invertible. (iii) Riesz idempotent. The Riesz idempotent $E_{\lambda}(T)$ associated with a nonzero isolated spectral point λ is self-adjoint under some assumptions. (iv) Approximate point spectrum and orthogonal eigen-spaces. T has the spectral property (II-1). Meanwhile some examples are given: (i) There exists an operator T such that T is $(n+1)$-paranormal, T is not n-paranormal, T^{-1} is not normaloid and T^* is not m-paranormal for every positive integer m. (ii) There exists an operator T such that T is $(n,1)$-quasiparanormal, T is not n-paranormal, $\mathscr{M} = \ker T \neq \{0\}$ and $\ker T_{22} \neq \{0\}$.

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