Operators and Matrices
Volume 11, Number 3 (2017), 891–899
doi:10.7153/oam-11-63

ON DECOMPOSITION OF OPERATORS HAVING Γ_3 AS A SPECTRAL SET

SOURAV PAL

Abstract. The symmetrized polydisc of dimension three is the set

$$\Gamma_3 = \{(z_1+z_2+z_3, z_1z_2+z_2z_3+z_3z_1, z_1z_2z_3) : |z_i| \leqslant 1, i = 1,2,3\} \subseteq \mathbb{C}^3.$$

A triple of commuting operators for which Γ_3 is a spectral set is called a Γ_3-contraction. We show that every Γ_3-contraction admits a decomposition into a Γ_3-unitary and a completely non-unitary Γ_3-contraction. This decomposition parallels the canonical decomposition of a contraction into a unitary and a completely non-unitary contraction. We also find new characterizations for the set Γ_3 and Γ_3-contractions.

Mathematics subject classification (2010): 47A13, 47A15, 47A20, 47A25, 47A45.
Keywords and phrases: Spectral set, symmetrized tridisc, Γ_3-contraction, canonical decomposition.

REFERENCES

[1] J. AGLER AND N. J. YOUNG, *A commutant lifting theorem for a domain in $\mathbb{C}^2$ and spectral interpolation*, J. Funct. Anal. **161** (1999), 452–477.
[2] J. AGLER AND N. J. YOUNG, *A model theory for Γ-contractions*, J. Operator Theory **49** (2003), 45–60.
[3] J. AGLER AND N. J. YOUNG, *Operators having the symmetrized bidisc as a spectral set*, Proc. Edin. Math. Soc. **43** (2000), 195–210.
[4] R. BHATIA, *Positive definite matrices*, Princeton Series in Applied Mathematics, Princeton University Press, Princeton, NJ, 2007.
[5] T. BHATTACHARYYA, S. PAL AND S. SHYAM ROY, *Dilations of Γ-contractions by solving operator equations*, Adv. Math. **230** (2012), 577–606.
[6] T. BHATTACHARYYA AND S. PAL, *A functional model for pure Γ-contractions*, J. Operator Thoery **71** (2014), 327–339.
[7] S. BISWAS AND S. SHYAM ROY, *Functional models for Γ_n-contractions and characterization of Γ_n-isometries*, J. Func. Anal. **266** (2014), 6224–6255.
[8] H. BERCOVICI, C. FOIAS, L. KERCHY AND B. SZ.-NAGY, *Harmonic analysis of operators on Hilbert space*, Universitext, Springer, New York, 2010.
[9] C. COSTARA, *On the spectral Nevanlinna-Pick problem*, Studia Math. **170** (2005), 23–55.
[10] A. EDIGARIAN AND W. ZWONEK, *Geometry of symmetrized polydisc*, Archiv der Mathematik **84** (2005), 364–374.
[11] S. PAL, *From Stinespring dilation to Sz.-Nagy dilation on the symmetrized bidisc and operator models*, New York Jour. Math. **20** (2014), 545–564.
[12] S. PAL, *Rational dilation on the symmetrized tridisc: falire, success and unknown*, `https://arxiv.org/abs/1610.00425`.
[13] S. PAL AND O. M. SHALIT, *Spectral sets and distinguished varieties in the symmetrized bidisc*, J. Funct. Anal. **266** (2014), 5779–5800.
[14] J. VON NEUMANN, *Eine Spektraltheorie für allgemeine Operatoren eines unitären Raumes*, Math. Nachr. **4** (1951), 258–281.

Paper OaM-11-63