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COMPLEX SYMMETRIC OPERATORS, SKEW SYMMETRIC OPERATORS AND REFLEXIVITY

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Abstract. Let $\mathscr{H}$ be a complex separable infinite-dimensional Hilbert space and C be a conjugation on $\mathscr{H}$. Let $\mathscr{C}$ and $\mathscr{S}$ denote respectively the set of C-symmetric operators and the set of C-skew-symmetric operators on $\mathscr{H}$. It is proved that $\mathscr{C}$ and $\mathscr{S}$ are Roberts orthogonal to each other, and some distance formulas from an operator to the sets $\mathscr{C}$, $\mathscr{S}$ are obtained. We exhibit the annihilating relation between $\mathscr{C}$ and $\mathscr{S}$ by describing their preannihilators. As applications, it is shown that $\mathscr{S}$ is hyperreflexive and not transitive.

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REFERENCES

[1] G. BIRKHOFF, *Orthogonality in linear metric spaces*, Duke Math. J. **1** (1935), no. 2, 169–172.
[2] J. B. CONWAY, *A course in operator theory*, Graduate Studies in Mathematics, 21. American Mathematical Society, Providence, RI, 2000.
[3] S. R. GARCIA, *Approximate antilinear eigenvalue problems and related inequalities*, Proc. Amer. Math. Soc. **136** (2008), no. 1, 171–179 (electronic).
[4] S. R. GARCIA AND D. E. POORE, *On the norm closure problem for complex symmetric operators*, Proc. Amer. Math. Soc. **141** (2013), no. 2, 549–549.
[5] S. R. GARCIA AND D. E. POORE, *On the norm closure of the complex symmetric operators: compact operators and weighted shifts*, J. Funct. Anal. **264** (2013), no. 3, 691–712.
[6] S. R. GARCIA, E. PRODAN, AND M. PUTINAR, *Mathematical and physical aspects of complex symmetric operators*, J. Phys. A: Math. Gen. **47** (2014), no. 35, 353001.
[7] S. R. GARCIA AND M. PUTINAR, *Complex symmetric operators and applications*, Trans. Amer. Math. Soc. **358** (2006), no. 3, 1285–1315 (electronic).
[8] S. R. GARCIA AND M. PUTINAR, *Complex symmetric operators and applications, II*, Trans. Amer. Math. Soc. **359** (2007), no. 8, 3913–3931 (electronic).
[9] S. R. GARCIA AND W. R. WOGEN, *Complex symmetric partial isometries*, J. Funct. Anal. **257** (2009), no. 4, 1251–1260.
[10] S. R. GARCIA AND W. R. WOGEN, *Some new classes of complex symmetric operators*, Trans. Amer. Math. Soc. **362** (2010), no. 11, 6065–6077.
[11] K. GUO, Y. JI, AND S. ZHU, *A C^*-algebra approach to complex symmetric operators*, Trans. Amer. Math. Soc. **367** (2015), no. 10, 6903–6942.
[12] K. GUO AND S. ZHU, *A canonical decomposition of complex symmetric operators*, J. Operator Theory **72** (2014), no. 2, 529–547.
[13] R. A. HORN AND C. R. JOHNSON, *Matrix analysis*, Corrected reprint of the 1985 original, Cambridge University Press, Cambridge, 1990.
[14] K. KLIŚ-GARLICKA AND M. PTAK, *k-hyperreflexive subspaces*, Houston J. Math. **32** (2006), no. 1, 299–313 (electronic).
[15] K. KLIŚ-GARLICKA AND M. PTAK, *C-symmetric operators and reflexivity*, Oper. Matrices **9** (2015), no. 1, 225–232.

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[16] C. G. LI AND T. T. ZHOU, *Skew symmetry of a class of operators*, Banach J. Math. Anal. **8** (2014), no. 1, 279–294.
[17] C. G. LI AND S. ZHU, *Skew symmetric normal operators*, Proc. Amer. Math. Soc. **141** (2013), no. 8, 2755–2762.
[18] B. D. ROBERTS, *On the geometry of abstract vector spaces*, Tôhoku Math. J. **39** (1934), 42–59.
[19] S. M. ZAGORODNYUK, *On a J-polar decomposition of a bounded operator and matrices of J-symmetric and J-skew-symmetric operators*, Banach J. Math. Anal. **4** (2010), no. 2, 11–36.
[20] S. M. ZAGORODNYUK, *On the complex symmetric and skew-symmetric operators with a simple spectrum*, Symmetry, Integrability and Geometry: Methods and Applications **7** (2011), 1–9.
[21] S. ZHU, *Approximate unitary equivalence to skew symmetric operators*, Complex Anal. Oper. Theory **8** (2014), no. 7, 1565–1580.
[22] S. ZHU, *Skew symmetric weighted shifts*, Banach J. Math. Anal. **9** (2015), no. 1, 253–272.
[23] S. ZHU, *Approximation of complex symmetric operators*, Math. Ann. **364** (2016), no. 1-2, 373–399.
[24] S. ZHU AND C. G. LI, *Complex symmetry of a dense class of operators*, Integral Equations Operator Theory **73** (2012), no. 2, 255–272.
[25] S. ZHU, C. G. LI, AND Y. Q. JI, *The class of complex symmetric operators is not norm closed*, Proc. Amer. Math. Soc. **140** (2012), no. 5, 1705–1708.
[26] S. ZHU AND J. ZHAO, *The Riesz decomposition theorem for skew symmetric operators*, J. Korean Math. Soc. **52** (2015), no. 2, 403–416.