

## TRUNCATED MOMENT PROBLEMS IN $\mathbb{R}^2$ AND RECURSIVENESS

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*Abstract.* This paper concerns the study of the truncated moment problems in  $\mathbb{R}^2$ . More precisely, some Curto-Fialkow's results, on the truncated complex moment problems, combined with properties of generalized Fibonacci sequences, allow us to establish some results on the truncated moment problems in  $\mathbb{R}^2$  and its extension to the full moment problems. Furthermore, with the aid of the combinatorial formula of the generalized Fibonacci sequences, we infer a combinatorial expression for each term of the associated moment matrix. Which in turn permits us to construct a process for expressing the terms of the extension of the truncated moment problem in  $\mathbb{R}^2$  to the full moment problems. As a consequence, we are able to provide a detailed study of the truncated quartic moment problem in  $\mathbb{R}^2$ , linked firmly to the associated moment matrix. Illustrative examples and applications are given.

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